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A thorough investigation of the interdependent behavior of prestressed system

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ABSTRACT

Due to the existing of many prestressed members in the structural system, the interdependent behavior of all prestressed members is the main concern in the analysis of the pretension process. A thorough investigation of this mutual effect is essential for an effective, reliable, and optimal analysis. Focus on this aspect, this paper presents an investigation of the interdependent behavior of all prestressed members in the whole structural system based on influence matrix (IFM). Four different types of IFM are introduced. Two different solving methods are brought forth to analyze the pretension process. The direct solving method solves for the accurate solution, whereas the iterative solving method repeatedly amends to achieve an approximate solution. A numerical example is then conducted. The result shows that various kinds of complicated batched and repeated tensioning schemes can be analyzed reliably, effectively, and completely based on IFM.

Keywords: Finite element analysis, influence matrix, interdependent behavior, pretension process

1. INTRODUCTION

In recent decades, prestress has been widely applied in space structures due to the capacity to improve the structural rigidity, reduce the structural deformation and redistribute internal force among structural members. Therefore, prestressed structures can cover a larger span, have smaller structural weight, and become more slender (Levy et al., 1994). As a result, more economic structures can be achieved.

Due to the interdependent behavior of all prestressed members in the system, as one member is tensioned, the forces of other structural members will immediately change. Therefore, the main concern is how to predict the required tensioning control forces and/or displacements needed to apply upon each prestressed member in order to finally meet the design requirement for a specific design stage or the so called the target state. In order words, once the predicted tensioning control forces and/or displacements are applied on each prestressed member according to the predetermined construction scheme, the forces in prestressed members in the target state must reach the target forces; the nodal displacements of prestressed members in the target state must reach the target displacements after finish tensioning. Moreover, because of the presence of many prestressed members in the structural system and the limited capability of tensioning equipment, it is impossible to tension all prestressed members at the same time. Therefore, the batched and repeated tensioning schemes are unavoidable. This makes the analysis of the whole pretension process, is quite difficult and rather important. Focusing on this aspect, there are two important problems arisen. The first problem is how to predict the required tensioning control forces and/or displacements instead of blindly and endlessly supplemental tension. This problem needs to be solved beforehand in the design stage and usually based on the theoretical structural model. The second problem is there are often existed influence factors that make the actual structural state in construction somewhat different with the theoretical model such as simplify assumptions of the theoretical model, fabricated errors, temperature loads, friction of structural components. That makes the design prestressed state could not be achieved even though the pretension scheme analyzed beforehand has already been followed. As a result, the actual tensioning control forces and/or displacements in the construction stage need to be reanalyzed.

Aimed at solving the first problem, Zhuo and Ishikawa (2004) proposed the ‘tensile force compensation analysis method’ to find the tensioning control force for hybrid structure in which prestressed member is assembled and tensioned one by one. This method requires a large amount of calculation due to the necessity to repeatedly analyze the whole structure under the subsequent compensation tensile forces. Later, Dong and Yuan (2007) introduced the ‘initial internal force method’ that studied the mutual relation between prestressed members using flexibility method to analyze the pretension process of prestressed space grid structures. This method could study the batched tension though the repeated tension scheme was not addressed. In 2010s, Zhou et al. (2010b) presented two methods to study the pretension process of arch supported prestressed grid structures. The interaction behavior is studied by the mixed influence matrix based on displacement method, capable of studying different pretension schemes. On the other hand, the iterative approximation method repeatedly searched for an approximate solution without a clear understanding of the interaction between structural members. As a result, extremely slow convergence or even no convergence was noticed in some cases (Nguyen & Iu, Unpublised). Later, Zhou et al. (2014) combined an iterative method for form finding and the sequential analysis method (Zhou et al., 2010b) for pretension process simulation of suspen-dome structures. At the same time, He et al. (2011) provided a method to calculate the initial strains of cables to meet the design requirements also by iteration. This method in basic is similar with the ‘tensile force compensation analysis method’ (Zhuo & Ishikawa, 2004) and is applied for prestressed space reticulated structures. More recently, Li et al. (2014) proposed a nonlinear simulation analysis using cyclic iteration method for cable-supported barrel shell structures. As already mentioned, the main drawback of the iterative method is slow convergence or even no convergence was noticed in some cases(Nguyen & Iu, Unpublised).

Aimed at solving the second problem, Zhuo et al. (2008) presented the ‘tensile force correction calculation method’. However, this new method is in basic the same with the ‘tensile force compensation analysis method’ (Zhuo & Ishikawa, 2004). It also based on the structural analysis of the same theoretical model. Hence, the solution is not reliable in case there is a large difference between the theoretical and the actual structural model. Later Zhou et al. (2010c) introduced the ‘pretension scheme decision analysis method’ using an iterative calculation method based on the recursive of cable forces to solve for an approximate solution (Zhou et al., 2010a). If high accuracy is required, a large number of iteration is unavoidable, especially when there are many prestressed members in the structure. Besides extremely slow convergence or even no convergence was noticed in some particular cases (please refer to section 4).

Overall, most of the aforementioned methods remain some drawback. Therefore, a reliable, effective, and complete analysis of the whole pretension process is still needed. To this end, this paper presents a thorough investigation of the interdependent behavior of all prestressed members in the whole structural system based on influence matrix (IFM). Once the IFM is established, the complete analysis for the whole pretension process can be obtained.

In summary, IMF concept is introduced in section 2. Different types of IFM such as force based IFM, displacement based IFM, displacement-force based IFM, and force-displacement based IFM together with their characteristics are also addressed. Section 3 brings forth the application of IFM in the analysis of pretension process as well as the direct solving method and the iterative solving method. A numerical example that investigates the interdependent behavior of all prestressed members in the whole system during the pretension process is then established in Section 4. Different pretension schemes are illustrated. Both batched tension and repeated tension are considered, such as each prestressed member is assembled and tensioned one by one; many members are assembled and tensioned in batches; all members are assembled and then tensioned simultaneously as well as all members are assembled first and then tensioned in multistep. A detailed discussion follows afterwards. Finally, some conclusions are obtained in section 5. The result shows that this approach is reliable, efficient, and complete.

2. INFLUENCE MATRIX

2.1 Influence matrix concept

IFM represents the mutual influence of forces and/or displacements of prestressed members in the whole structure.

$$M = \begin{bmatrix} m_{11} & m_{12} & .. & m_{1n} \\ m_{21} & m_{22} & .. & m_{2n} \\ .. & .. & .. & .. \\ m_{n1} & m_{n2} & .. & m_{nn} \end{bmatrix} \tag{1}$$

In which matrix coefficient m_{kj} is the force or displacement variable quantity of member k once the force or displacement of member j increases by one unit; n is the number of prestressed members.

2.2 Different types of IFM

The force based IFM, A matrix, the displacement based IFM, B matrix, the displacement-force based IFM, C matrix, and the force-displacement based IFM, D matrix, are four different types of IFM that represent the interdependent of member forces and/or displacements during pretension process. They can be categorized into two different groups: one-criterion IFM, A and B matrix, which controls force or displacement and two-criterion IFM, C and D matrix, which controls both force and displacement.

The coefficient a_{kj} of A matrix is the force variation of member k when the force of member j is increased by one unit; the coefficient b_{kj} of B matrix is the displacement variation of member k when the displacement of member j is increased by one unit; the coefficient c_{kj} of C matrix is the displacement variation of member k when the force of member j is increased by one unit and the coefficient d_{kj} of D matrix is the force variation of member k when the displacement of member j is increased by one unit.

IFMs of prestressing forces and/or displacements are system state parameters. They can be obtained from finite element analysis based on the formulation proposed by Iu and Bradford (2012a), (2012b). More specifically, there are two different ways to establish the IFM: whether using the concept of prestress by lack of fit (reader may refer to previous literature (Felton & Dobbs, 1977; Felton & Hofmeister, 1970; Hanaor & Levy, 1985; Levy & Hanaor, 1992; Spillers & Levy, 1984; Zhou et al., 2010b) for this concept) or from structural analysis simulating the construction sequence.

In case the IFM constructed by imposing -1 unit lack of fit upon each prestressed member, the IFM coefficients of A, B, C, and D matrices are:

$$a_{kj} = \frac{f'_{jk}}{f'_{jj}}, b_{kj} = \frac{\delta'_{jk}}{\delta'_{jj}}, c_{kj} = \frac{\delta'_{jk}}{f'_{jj}}, d_{kj} = \frac{f'_{jk}}{\delta'_{jj}} \tag{2}$$

In which f'_{jk} and δ'_{jk} is the force and displacement of member k under -1 unit lack of fit of member j.

In case the IFM constructed based on the member forces and/or displacements from different stages of the construction sequence, they are:

$$\begin{aligned} a^i_{kj} &= \frac{f^i_{jk} - f^i_{j-i,k}}{f^i_{jj} - f^i_{j-1,j}}, b^i_{kj} = \frac{\delta^i_{jk} - \delta^i_{j-i,k}}{\delta^i_{jj} - \delta^i_{j-1,j}} \\ c^i_{kj} &= \frac{\delta^i_{jk} - \delta^i_{j-i,k}}{f^i_{jj} - f^i_{j-1,j}}, d^i_{kj} = \frac{f^i_{jk} - f^i_{j-i,k}}{\delta^i_{jj} - \delta^i_{j-1,j}} \end{aligned} \tag{3}$$

In which f_{jk}^{i} and δ_{jk}^{i} is the force and displacement of member *k* after member *j* is tensioned for the i^{th} round.

2.3 The characteristic of IFM

1. All the coefficients in the diagonal of the one-criterion IFM, *A* and *B* matrices, are equal to 1 ($a_{kj} = 1, b_{kj} = 1$ with *k* = *j*).

2. The IFMs depend on the structural stiffness. If the structural stiffness changes according to the construction sequence, the IFMs will also change. Under different pretension schemes such as each prestressed member is assembled and tensioned in turn, many members are assembled and tensioned in batches or all members are assembled first and then tensioned, the IFMs are all different.
   a. In case, each member is assembled and tensioned one by one: the coefficients of *A* and *D* matrices, $a_{kj} = 0, d_{kj} = 0$ with *k* > *j*. The reason is when member *j* is assembled and tensioned, member *k* with *k* > *j* is still not assembled. Therefore, *A* and *D* have the form of an upper triangle. In contrast, $b_{kj} \neq 0, c_{kj} \neq 0$ with *k* > *j*, as *B* and *C* matrices represent the mutual relation of nodal displacements instead of member forces.
   b. In case many members are assembled and tensioned in batches, the same characteristic as in case (a) can be seen. Column *j* of IFMs, in this case, represents the force or displacement variable quantity of batch *k* once the force or displacement of batch *j* increases by one unit. That means batch of members is considered instead of an individual member.
   c. In case, all members are assembled first and then tensioned, all the IFM coefficients $a_{kj} \neq 0, b_{kj} \neq 0$ $c_{kj} \neq 0, d_{kj} \neq 0$ with *k* > *j*.

3. The determinant of IFM is an important parameter that controls which solving method should be used for the pretension process analysis (a detailed discussion is given in section 4).

3. THE PRETENSION PROCESS ANALYSIS BASED ON IFM

Once the IFM has been constructed, the required initial lack of fit as well as the required tensioning control forces and/or displacements in order to achieve the design requirement for a specific prestressed state can be determined using the direct solving method or the iterative solving method as follows:

3.1 The direct solving method

With the definition of the IFM, the explicit relation between the required initial lack of fit to achieve the target forces and or displacements at a particular stage can be simulated in matrix form as follows:

$$\begin{aligned} &A.\overline{A}.L = F_t \\ &B.\overline{B}.L = \Delta_t \\ &C.\overline{A}.L = \Delta_t \\ &D.\overline{B}.L = F_t \end{aligned} \tag{4}$$

In which $\overline{A}$ and $\overline{B}$ are diagonal matrices as in (5); *L* is the required lack of fit vector; F_t and Δ_t are the target force and displacement vector respectively for a specific prestressed state. Column *j* of the productive matrix $A.\overline{A}$ and $D.\overline{B}$, $B.\overline{B}$ and $C.\overline{A}$ represent the structural member forces and nodal displacements respectively under -1 unit initial lack of fit of member *j* (Zhou et al., 2010b).

$$\overline{A} = \begin{bmatrix} f_{11}^{'} & 0 & .. & 0 \\ 0 & f_{22}^{'} & .. & 0 \\ .. & .. & .. & .. \\ 0 & 0 & .. & f_{nn}^{'} \end{bmatrix}, \overline{B} = \begin{bmatrix} \delta_{11}^{'} & 0 & .. & 0 \\ 0 & \delta_{22}^{'} & .. & 0 \\ .. & .. & .. & .. \\ 0 & 0 & .. & \delta_{nn}^{'} \end{bmatrix} \tag{5}$$

Simply inverse these productive matrices, Eq. (4) can be directly solved for the required lack of fit *L*. Then *L* can be imposed directly on the corresponding member, by back substitution *L* in Eq. (4), considering the construction sequence. The member forces and/or nodal displacements during the pretension process as well as the tensioning control forces and/or displacements can then be all obtained.

3.2 The iterative solving method

With the definition of the IFM, the mutual relation between member forces and/or displacements during pretension process can be simulated in matrix form (Zhou et al., 2010a) as follows:

$$\begin{aligned} &F_j^i = X_j^i A_j^i + F_{j-1}^i \\ &\Delta_j^i = Y_j^i B_j^i + \Delta_{j-1}^i \\ &\Delta_j^i = X_j^i C_j^i + \Delta_{j-1}^i \\ &F_j^i = Y_j^i D_j^i + F_{j-1}^i \end{aligned} \tag{6}$$

In which for the *i*th round tension: $F_j^i = [f_{j1}^i, f_{j2}^i .. f_{jn}^i]^T$, $\Delta_j^i = [\delta_{j1}^i, \delta_{j2}^i .. \delta_{jn}^i]^T$ is the member force, displacement vector respectively when the j^{th} member is tensioned; With *j = 1*, F_{j-1}^i and Δ_{j-1}^i are the member force and displacement vector before pretension; $X_j^i = t_j^i - f_{j-1,j}^i$ and $Y_j^i = u_j^i - \delta_{j-1,j}^i$ are the incremental force and displacement respectively of the j^{th} member; t_j^i and u_j^i are the tensioning control force and displacement of member *j*; $A_j^i = [a_{1j}^i, a_{2j}^i .. a_{nj}^i]^T$, $B_j^i = [b_{1j}^i, b_{2j}^i .. b_{nj}^i]^T$,

$C_j^i = [c_{1j}^i, c_{2j}^i..c_{nj}^i]^T$, $D_j^i = [d_{1j}^i, d_{2j}^i..d_{nj}^i]^T$ are the column *j* of *A*, *B* , *C*, and *D* matrices respectively.

The required tensioning control forces and/or displacements to achieve the target prestressed state can be determined by solving Eq. (7) as follows:

$$\begin{aligned} F_n^i &= X_j^i A_j^i + F_{j-1}^i = F_t^i \\ \Delta_n^i &= Y_j^i B_j^i + \Delta_{j-1}^i = \Delta_t^i \\ \Delta_n^i &= X_j^i C_j^i + \Delta_{j-1}^i = \Delta_t^i \\ F_n^i &= Y_j^i D_j^i + F_{j-1}^i = F_t^i \end{aligned} \tag{7}$$

In which for the i^{th} round tension: F_n^i and Δ_n^i are member force and displacement vector after finish tensioning *n* prestressed members; F_t^i and Δ_t^i are the target force and displacement vector respectively.

For example, in case there are two prestressed members tensioned one time only, the set of Eq. (7) based on *A* and *B* matrices are respectively as in Eq. (8) and (9).

$$\begin{aligned} F_1 &= [f_{11} \quad f_{12}]^T = (t_1 - f_{01})[a_{11} \quad a_{21}]^T + [f_{01} \quad f_{02}]^T \\ F_2 &= [f_{21} \quad f_{22}]^T = (t_2 - f_{12})[a_{12} \quad a_{22}]^T + [f_{11} \quad f_{12}]^T = [f_{t1} \quad f_{t2}]^T \end{aligned} \tag{8}$$

$$\begin{aligned} \Delta_1 &= [\delta_{11} \quad \delta_{12}]^T = (u_1 - \delta_{01})[b_{11} \quad b_{21}]^T + [\delta_{01} \quad \delta_{02}]^T \\ \Delta_2 &= [\delta_{21} \quad \delta_{22}]^T = (u_2 - \delta_{12})[b_{12} \quad b_{22}]^T + [\delta_{11} \quad \delta_{12}]^T = [\Delta_{t1} \quad \Delta_{t2}]^T \end{aligned} \tag{9}$$

By back substitute F_1 into F_2 in Eq. (8); Δ_1 into Δ_2 in Eq. (9), the required tensioning control forces and/or displacements can be obtained by directly solving this set of two equations. The direct solutions of Eq. (8) and (9) are given in the appendix.

It is worth to mention that if there are a large number of prestressed members in the structures, directly solving the set of Eq. (7) is rather complicated. In this case, the iterative solving method is more preferable (Zhou et al., 2010c).

3.1.1 The iterative solving procedure

Step1. Let $i = 1$, the tensioning control forces and/or displacements are set equal to the target forces and/or displacements at the beginning.

$$T^i = F_t, U^i = \Delta_t \tag{10}$$

In which T^i and U^i are the tensioning control force and displacement vector respectively.

Step2. The member forces and displacements after finish tension F_n^i and Δ_n^i can be computed using Eq. (6).

Step3. Check the deviation between the computed forces and/or displacements with the target values as in Eq. (11).

$$\varepsilon_f = 1 - \frac{F_n^i}{F_t},\ \varepsilon_\delta = 1 - \frac{\Delta_n^i}{\Delta_t} \tag{11}$$

If the deviation is smaller than a required tolerance, these tensioning control forces and/or displacements are the expected values. Otherwise, they need to be adjusted by adding up with the deviation values as in Eq. (12) and return to step2 for the $(i+1)^{th}$ iteration.

$$T^{i+1} = T^i + (F_n^i - F_t),\ \Delta^{i+1} = U^i + \left(\Delta_n^i - \Delta_t\right) \tag{12}$$

The procedure is repeated until convergence is detected.

4. A NUMERICAL EXAMPLE OF THE INTERDEPENDENT BEHAVIOR OF PRESTRESSED SYSTEM

4.1 Design requirements and pretension schemes

A steel frame column supporting compression load from roof panel and glass façade is as Fig.1. The main frame is tube 100x100x10mm, f_y = 355 KN/mm^2 (the material nonlinearity is not considered) and E = 210x10^{6}KN/m^2; The prestressed member is 70x35x4mm, f_y = 550 KN/mm^2 and E = 210x10^{6}KN/m^2. The target forces are 20KN in prestressed members No.**11**, **14** and 10KN in members No.**12**, **13**. The target horizontal displacements at nodes No.**10**, **7** are -29mm and nodes No.**9**, **8** are -61.7mm (inward displacement) at the end.

Considering the structural properties, the design requirements and the capability of the tensioning equipment, four pretension schemes are proposed as follows:

1. **Scheme A***:* After constructing the main frame, the prestressed member is assembled and tensioned from member No.**11** to **14** in turn. The compression load 100KN is then applied on the column.
2. **Scheme B***:* The construction sequence is the same as in scheme A. However, the prestressed members are assembled and prestressed in batches. Batch No.1 included member No.**11** and **14** are assembled and tensioned first. Then batch No.2 included member No.**12** and **13** are assembled and tensioned later.
3. **Scheme C***:* After constructing the main frame, all the prestressed members are assembled and prestressed simultaneously. The compression load is then applied.
4. **Scheme D***:* After constructing the main frame, all the prestressed members are assembled first and then prestressed one by one to 10KN in members No.**11** and **14** and 5KN in members No.**12** and **13**. After applying the compression load, the second tensioning phase is conducted to achieve the final design requirements.

4.2 Analysis of the whole pretension process

Under **scheme A**, the prestressed member is assembled and tensioned one by one. Under **scheme B**, they are assembled and tensioned batches by batches. Therefore, the structural stiffness is changed after each tensioning step. On the other hand, under **schemes C** and **D**, all the prestressed members are assembled first and then tensioned, the structural stiffness remains unchanged. The prestressing forces and displacements are obtained from the finite element analysis programmed in FORTRAN language based on the formulation of lu and Bradford (2012a),(2012b). The direct solving method and the iterative solving method are realized on Visual Basic platform.

For the sake of simplicity, only the final analysis results of the prestressed members are given. The IFMs are first constructed. Then the required lack of fit to achieve the target design forces and/or displacements are determined according to Eq. (4) as in Table 1. The force and displacement variations during pretension process as well as the tensioning control forces and displacements (the bold value) analyzed by the direct solving method using Eq. (4) and the iterative solving method using Eq. (6) are all addressed in Tables 2 and 3. It can be seen that the target forces (the underline values) are all achieved at the end based on this approach. Besides, it is worth to mention that these two solving methods are capable of yielding the same solution as in this numerical example.

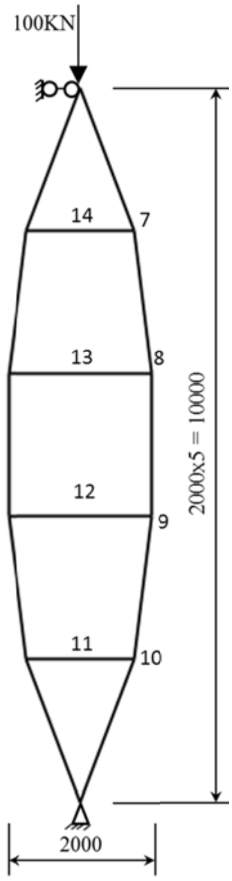

Fig. 1 The structural model of the frame column

IFMs under different pretension schemes are as follows:

Scheme A:

$$A=\begin{bmatrix}1 & -0.281 & 0.342 & 0.067\\ 0 & 1 & -0.407 & 0.092\\ 0 & 0 & 1 & -0.242\\ 0 & 0 & 0 & 1\end{bmatrix},\ B=\begin{bmatrix}1 & 0.407 & 0.318 & 0.411\\ 1.466 & 1 & 0.785 & 1.012\\ 1.060 & 0.872 & 1 & 1.295\\ 0.370 & 0.328 & 0.459 & 1\end{bmatrix}$$

$$C=\begin{bmatrix}-0.0015 & -0.0017 & -0.0012 & -0.0005\\ -0.0021 & -0.0043 & -0.0029 & -0.0011\\ -0.0016 & -0.0037 & -0.0037 & -0.0015\\ -0.0005 & -0.0014 & -0.0017 & -0.0011\end{bmatrix},\ D=\begin{bmatrix}-682.7 & 65.9 & -92.3 & -59.7\\ 0 & -234.6 & 110.0 & -81.5\\ 0 & 0 & -270.0 & 215.1\\ 0 & 0 & 0 & -889.5\end{bmatrix}$$

Scheme B:

$$A=\begin{bmatrix}1 & -0.734\\ 0 & 1\end{bmatrix},\ B=\begin{bmatrix}1 & 0.353\\ 1.843 & 1\end{bmatrix}$$

$$C=\begin{bmatrix}-0.0020 & -0.0022\\ -0.0037 & -0.0063\end{bmatrix},\ D=\begin{bmatrix}-498.178 & 116.322\\ 0 & -158.553\end{bmatrix}$$

Scheme C and D:

$$A=\begin{bmatrix}1 & -0.799 & 0.317 & -0.067\\ -0.634 & 1 & -0.738 & 0.251\\ 0.251 & -0.738 & 1 & -0.634\\ -0.067 & 0.317 & -0.799 & 1\end{bmatrix},\ B=\begin{bmatrix}1 & 0.006 & -0.003 & 0.001\\ 0.008 & 1 & 0.008 & -0.003\\ -0.003 & 0.008 & 1 & 0.008\\ 0.001 & -0.003 & 0.006 & 1\end{bmatrix}$$

$$C=\begin{bmatrix}-0.0004579 & -0.0000037 & 0.0000015 & -0.0000003\\ -0.0000039 & -0.0005766 & -0.0000045 & 0.0000015\\ 0.0000015 & -0.0000045 & -0.0005766 & -0.0000039\\ -0.0000003 & 0.0000015 & -0.0000037 & -0.0004579\end{bmatrix}$$

$$D=\begin{bmatrix}-2184.1 & 1385.0 & -549.2 & 146.0\\ 1384.2 & -1734.3 & 1279.8 & -548.9\\ -548.9 & 1279.8 & -1734.3 & 1384.2\\ 146.0 & -549.2 & 1385.0 & -2184.1\end{bmatrix}$$

Table 1. The required lack of fit (in m)

Member	11	12	13	14
Scheme A	0.0197	0.0183	0.0131	0.0071
Scheme B	0.0417	0.0165	0.0165	0.0417
Scheme C	0.0582	0.1235	0.1235	0.0582
Scheme D - Step 1	0.0772	0.1642	0.1642	0.0772
Scheme D - Step 2	-0.0190	-0.0407	-0.0407	-0.0190

Table 2. Forces of prestressed members in various tensioning phases (KN)

Scheme A	Member	11	12	13	14
Assemble and tension member:	11	**6.71**			
	12	5.23	**5.27**		
	13	7.13	3.01	**5.56**	
	14	7.64	3.71	3.71	**7.64**
Install roof system and glass façade		20.00	10.00	10.00	20.00

Scheme B	Batch	1	2
Assemble and tension batch:	1	**10.36**	
	2	7.64	**3.71**
Install roof system and glass façade		20.00	10.00

Scheme C	Member	11	12	13	14
Assemble and tension member simultaneously		**7.64**	**3.71**	**3.71**	**7.64**
Install roof system and glass façade		20.00	10.00	10.00	20.00

Scheme D	Member	11	12	13	14
First-time tension member:	11	**83.49**	-52.91	20.98	-5.58
	12	-29.05	**88.00**	-83.00	39.05
	13	15.58	-15.98	**57.91**	-73.49
	14	10.00	5.00	5.00	**10.00**
Install roof system and glass façade		22.36	11.29	11.29	22.36
Second-time tension member:	11	**1.78**	24.34	6.12	23.74
	12	29.70	**-10.62**	31.92	12.67
	13	18.63	15.18	**-3.04**	40.59
	14	20.00	10.00	10.00	**20.00**

Table 3. Horizontal displacements in various tensioning phases (mm)

Scheme A	Nodal	10	9	8	7
Assemble and tension member:	11	**-9.83**	-14.41	-10.42	-3.64
	12	-18.98	**-36.89**	-30.03	-11.02
	13	-25.53	-53.04	**-50.62**	-20.47
	14	-29.06	-61.74	-61.74	**-29.06**
Install roof system and glass façade		-29.01	-61.70	-61.70	-29.01

Scheme B	Nodal	10, 7	9, 8
Assemble and tension batch:	1	**-20.91**	-38.54
	2	-29.10	**-61.73**
Install roof system and glass façade		-29.04	-61.69

Scheme C	Nodal	10	9	8	7
Assemble and tension member simultaneously		**-29.06**	**-61.68**	**-61.68**	**-29.06**
Install roof system and glass façade		-29.01	-61.65	-61.65	-29.01

Scheme D	Nodal	10	9	8	7
First-time tension member:	11	**-38.23**	-0.32	0.13	-0.03
	12	-38.74	**-81.58**	-0.51	0.18
	13	-38.54	-82.22	**-81.76**	-0.34
	14	-38.56	-82.09	-82.09	**-38.56**
Install roof system and glass façade		-38.51	-82.05	-82.05	-38.51
Second-time tension member:	11	**-29.08**	-81.97	-82.08	-38.50
	12	-28.95	**-61.81**	-81.92	-38.55
	13	-29.01	-61.65	**-61.76**	-38.42
	14	-29.00	-61.68	-61.68	**-29.00**

4.3 Discussion

1. Using the proposed approach, the interdependent behavior of all prestressed members in the system is clarified. The tensioning control forces and displacements (the bold values in Tables 2 and 3) to meet the design requirements (the underline values) are all predicted. At the same time, the variation of member forces and displacements according to the construction sequence can also be obtained. These data can serve as a monitoring unit for the whole pretension process.

2. The pretension process analysis based on IFM approach using the direct solving method and the iterative solving method both achieve the design prestressed state. On the one hand, the direct solving method solves Eq. (4) directly for the required lack of fit as a basis to compute the accurate force and displacement variations during pretension process as well as the accurate required tensioning control forces and displacements. On the other hand, the iterative solving method based on Eq. (6) avoids directly solving the set of equation Eq. (7); it repeatedly amends the tensioning control forces and/or displacements to achieve an approximate solution to meet the design requirements.

3. It can be seen that the analysis results from both the direct solving method and the iterative solving method (once the control tolerances are set equal to zero) are the same. Hence, the reliability of these two approaches can be confirmed.

4. Obviously when the number of iteration increases, the result's accuracy of the iterative solving method will also increase. If high accuracy is desired, the convergent criteria can be set really close to or even zero.

5. The two-criterion IFMs prove to be more effective as compared with the one-criterion IFMs if both force and displacement are under control during the construction sequence and a monitoring unit for the whole pretension process is desired.

6. Based on the analysis result, the optimal pretension scheme can also be selected. Through the numerical example, the required tensioning control forces and displacements in **schemes A** and **C** are the smallest. However in **scheme C**, four prestressed members are required to be assembled and tensioned simultaneously. Therefore, high-performance instruments and automatic control technique with computer aid are necessary. This results in high construction cost. As a result, **scheme A** can be considered as the optimal pretension scheme. This confirms that this approach can serve as a platform to optimize the pretension process.

7. Due to the necessity to inverse the productive matrix, the direct solving method cannot be used if the determinant of the productive matrix is equal to zero. On the other hand, if the determinant of the IFM is close to zero, extremely slow convergence or even no convergence was noticed when using the iterative solving method. Therefore, the designer needs to understand the structural property or the IFM characteristic to decide which solving method will be used.

5. CONCLUSION

This paper presents a thorough investigation of the interdependent behavior of all prestressed members in the whole structural system that forms the basis for a complete analysis of the pretension process. Four different types of IFM and their application in the pretension process analysis are introduced. The direct and the iterative solving method are then proposed to analyze the pretension process.

It can be seen that once the IFM has been constructed, the required initial lack of fit as well as the tensioning control forces and/or displacements in order to meet the design requirement for a specific prestressed state can be obtained. Moreover the variation of prestressed member forces and nodal displacements according to the construction sequence can all be predicted. Therefore a thorough investigation of interdependent behavior of all prestressed members in the whole structural system is essential for a reliable, effective, and optimal pretension process analysis and the IFM proves to be quite suitable to fulfil this requirement. The numerical example shows that this approach can analyze various kinds of complicated batched and repeated tensioning schemes reliably and effectively.

In case, there is a large difference between the theoretical analytical model and the actual structural state, as a result, the IFM constructed by structural finite element analysis in the design stage no longer represent the actual interactions of the real structure. In this situation, the IFM need to be reconstructed based on the measured member forces and/or displacements using the monitoring unit with detective equipment in the construction stage. In this case, the proposed approach can also be used to obtain the correction of the tensioning forces and/or displacements in order to achieve the target prestressed state instead of supplemental tensions by trial and error. The basic differences are the system state parameter (IFM of the real structure) and the system initial value (the member forces and/or displacements that are measured after finishing the pretension phase following the predicted values in the design stage based on the theoretical model). Therefore, this approach is considered as complete and the above numerical example can serve as a benchmark investigation of the interdependent behavior of prestressed system.

APPENDIX

The tensioning control forces and displacements of the structure having two prestressed members are as follows:

$$
\begin{aligned}
X_1 &= t_1 - f_{01} = \frac{a_{22}\left(f_{t1} - f_{01}\right) - a_{12}\left(f_{t2} - f_{02}\right)}{a_{11}a_{22} - a_{21}a_{12}} \\
Y_1 &= u_1 - \delta_{01} = \frac{b_{22}\left(\Delta_{t1} - \delta_{01}\right) - b_{12}\left(\Delta_{t2} - \delta_{02}\right)}{b_{11}b_{22} - b_{21}b_{12}} \\
X_2 &= t_2 - f_{02} = -\frac{a_{21}\left(f_{t1} - f_{01}\right) - a_{11}\left(f_{t2} - f_{02}\right)}{a_{11}a_{22} - a_{21}a_{12}} + a_{21}\left(t_1 - f_{01}\right) \\
Y_2 &= u_2 - \delta_{02} = -\frac{b_{21}\left(\Delta_{t1} - \delta_{01}\right) - b_{11}\left(\Delta_{t2} - \delta_{02}\right)}{b_{11}b_{22} - b_{21}b_{12}} + b_{21}\left(u_1 - \delta_{01}\right)
\end{aligned}
\tag{13}
$$

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